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LETTER TO THE EDITOR

Parametrically controlling solitary wave dynamics in the modified Korteweg-de Vries equation

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Abstract
We demonstrate the control of solitary wave dynamics of the modified Korteweg-de Vries (MKdV) equation through the temporal variations of the distributed coefficients. This is explicated through exact cnoidal wave and localized soliton solutions of the MKdV equation with variable coefficients. The solitons can be accelerated and their propagation can be manipulated by suitable variations of the above parameters. In sharp contrast with the nonlinear Schrödinger equation, the soliton amplitude and widths are time independent.

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(Some figures in this article are in colour only in the electronic version)

The modified Korteweg-de Vries (MKdV) equation manifests in diverse areas of physics [1–6]. For example, it appears in the context of electromagnetic waves in size-quantized films, van Alfvén waves in collisionless plasma [7], phonons in anharmonic lattice [8], interfacial waves in two-layer liquid with gradually varying depth [9], transmission lines in Schottky barrier [10], ion acoustic solitons [11–13], elastic media [14] and traffic flow problems [15, 16]. It is an integrable dynamical system with an infinite number of conserved quantities; the solutions of this equation are well studied [17, 18]. Recently, the generation of solitons and modulational instability in this dynamical system has been carefully analysed [19].

Recently, nonlinear equations with variable coefficients have attracted considerable attention in the literature. The nonlinear Schrödinger equation (NLSE) with variable nonlinearity and dispersion is relevant to both optical fibres and Bose–Einstein condensates [20–23]. The nonlinear Schrödinger equation with source, having distributed coefficients such as variable dispersion, variable Kerr nonlinearity and gain or loss, is applicable to asymmetric twin-core optical fibres [24, 25]. It has been shown that solitons can be compressed and their dynamics effectively controlled through these variable parameters. The Korteweg-de Vries

(KdV) equation with variable coefficients [26] has been studied recently in the context of ocean waves, where the spatio-temporal variability of the coefficients is due to the changes in the water depth and other physical conditions. Recently, the extended KdV equation with variable nonlinearity has also been analysed for the effect of the varying coefficient on soliton dynamics [27]. The fact that the MKdV equation is relevant to hydrodynamics and a variety of physical phenomena, it is natural to expect the possibility of temporal variations in the equation parameters occurring in the same. Furthermore, for propagating solitons, the first integral of the MKdV equation yields NLSE with a source, making it imperative to investigate the effect of the temporal variation of the distributed parameters on the solitary wave solutions of this dynamical system.

The goal of the present letter is to study the effect of the variable coefficients on the solution space of the MKdV equation, both for positive and negative cases. We find that the effect of distributed coefficients on the soliton dynamics of the MKdV equation is quite different than that of the NLSE. In the case of the NLSE, the amplitude and width are affected by the time dependence of the distributed coefficients. This leads to compression of solitary waves in the NLSE. In the case of the MKdV equation, it is shown that solitary waves can be effectively controlled through the equation parameters. The solitons can be accelerated and manipulated by suitable variations of the above parameters. However, the width and amplitudes are not amenable for manipulation and control, unlike the NLSE system.

We consider the modified KdV equation with variable coefficients in the form

$$u_t + \alpha(t)u_x - \beta(t)u^2u_x + \gamma(t)u_{xxx} = 0, \tag{1}$$

where $\gamma(t)$, $\alpha(t)$ and $\beta(t)$ are time-dependent variables. Although the first derivative term in the field variable can be removed by a suitable change of the coordinate frame, the same has been kept here explicitly to contrast its effect with the nonlinear and dispersion terms. We consider the ansatz solution of the form

$$u = A_1(t)g[\omega(x,t)] + A_0(t), \tag{2}$$

where $\omega(x,t) = f(t)x - h(t)$. The variable coefficient MKdV equation can be mapped to Jacobi elliptic equation:

$$g'' = Pg + 2Qg^3, \tag{3}$$

having a conserved quantity

$$(g')^2 = Pg^2 + Qg^4. \tag{4}$$

Here, prime indicates differentiation with respect to the argument ω, and P and Q are constants. Substituting the ansatz into equation (1) and using relations (3) and (4), we collect the coefficients of g^α and $g^\alpha g'$ (where $\alpha = 0, 1, 2$) to find the consistency conditions

$$g^0: \quad \partial_t A_0 = 0, \tag{5}$$

$$g: \quad \partial_t A_1 = 0, \tag{6}$$

$$g': \quad A_1\partial_t\omega + A_1\alpha(t)\partial_x\omega + A_1P\gamma(t)[\partial_x\omega]^3 - A_0^2A_1\beta(t)\partial_x\omega = 0, \tag{7}$$

$$gg': \quad -2A_0A_1^2\beta(t)\partial_x\omega = 0, \tag{8}$$

$$g^2g': \quad -A_1^3\beta(t)\partial_x\omega + 6A_1Q\gamma(t)[\partial_x\omega]^3 = 0. \tag{9}$$

For obtaining non-trivial solutions to the above set of equations, we require that A_1, $\beta(t)$ and $\partial_x\omega$ are non-vanishing. Equation (8) then implies $A_0 = 0$; equation (6) yields $A_1 =$ constant. From equation (9), further simplification yields

$$f^2(t) = \frac{A_1^2\beta(t)}{6Q\gamma(t)}, \tag{10}$$

and from equation (7) we get

$$\partial_t h(t) = x\partial_t f(t) + \alpha(t) f(t) + P\gamma(t) f^3(t). \tag{11}$$

Condition (11) requires that $f(t)$ should be a constant so that the term containing x vanishes. This implies $\beta(t)/\gamma(t) = \kappa$, where κ is a constant. We then have the relations

$$f = \sqrt{\frac{A_1^2\kappa}{6Q}}, \tag{12}$$

and

$$h(t) = \sqrt{\frac{A_1^2\kappa}{6Q}} \int \left[\alpha(t) + \frac{\gamma(t)PA_1^2\kappa}{6Q}\right] \mathrm{d}t. \tag{13}$$

Hence, the exact travelling wave solution can be written in the form

$$u = A_1 g\left(\sqrt{\frac{A_1^2\kappa}{6Q}}\left[x - \int\left[\alpha(t) + \frac{\gamma(t)PA_1^2\kappa}{6Q}\right]\mathrm{d}t\right]\right). \tag{14}$$

It is worth noting that A_1 is unconstrained and controls the width of the solution. Unlike the case of the NLSE, the amplitude and width are independent of time. Since the solution involves κ, positive and negative MKdV equations have different types of solutions. g can be any of the 12 Jacobi elliptic functions with the modulus parameter m^2 ($0 \leqslant m^2 \leqslant 1$) [28, 29]. The following are some identities of the Jacobi elliptic functions which are used:

$$\begin{aligned}
&cn^2(w,m) + sn^2(w,m) = 1,\\
&dn^2(w,m) + m^2 sn^2(w,m) = 1,\\
&sn'(w,m) = cn(w,m)\cdot dn(w,m),\\
&cn'(w,m) = -sn(w,m)\cdot dn(w,m),\\
&dn'(w,m) = -m^2 sn(w,m)\cdot cn(w,m).
\end{aligned}$$

For $m = 1$,

$$cn(w,1) = dn(w,1) = \mathrm{sech}(w) \qquad \text{and} \qquad sn(w,1) = \tanh(w).$$

Below we analyse some explicit solutions and corresponding parameter ranges. For the sake of specificity, we consider $\beta(t) > 0$.

Case I

With $g = cn(\omega(x,t))$, one finds the cnoidal wave solution as

$$u = A_1 cn\left(\sqrt{\frac{A_1^2\kappa}{-6m^2}}\left[x - \int\left[\alpha(t) + \frac{\gamma(t)(2m^2-1)A_1^2\kappa}{-6m^2}\right]\mathrm{d}t\right]\right), \tag{15}$$

where $\gamma(t) < 0$, $P = (2m^2 - 1) > 0$, $Q = -m^2 < 0$ and $m^2 > 1/2$. In the case when $m^2 = 1/2$, $P = 0$ and $\gamma(t)$ does not affect the solution. For the case $m^2 = 1$, we have the exact solitary wave solution of the form

$$u = A_1 \mathrm{sech}\left(\sqrt{\frac{-A_1^2\kappa}{6}}\left[x - \int\left[\alpha(t) - \frac{\gamma(t)A_1^2\kappa}{6}\right]\mathrm{d}t\right]\right). \tag{16}$$

Figure 1 depicts the temporal evolution of the above bell-shaped localized solution. For illustrative purpose, we have considered two different cases, where $\alpha(t) = 0$ and

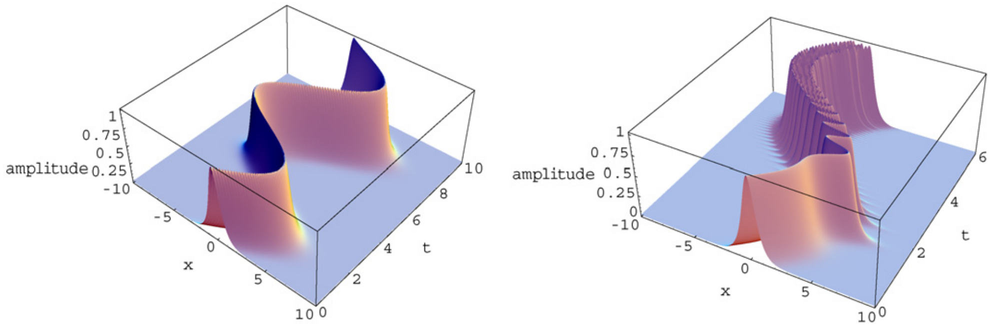

Figure 1. Propagating localized solitary wave solution of MKdV equation with $g = cn(x, t)$, where $\gamma = \cos(t)$, $\kappa = -24$, $A_1 = 1$, $P = 1$, $Q = -1$ and $m = 1$; left: $\alpha(t) = 0$ and right: $\alpha(t) = -3t^3\cos(t^3)$.

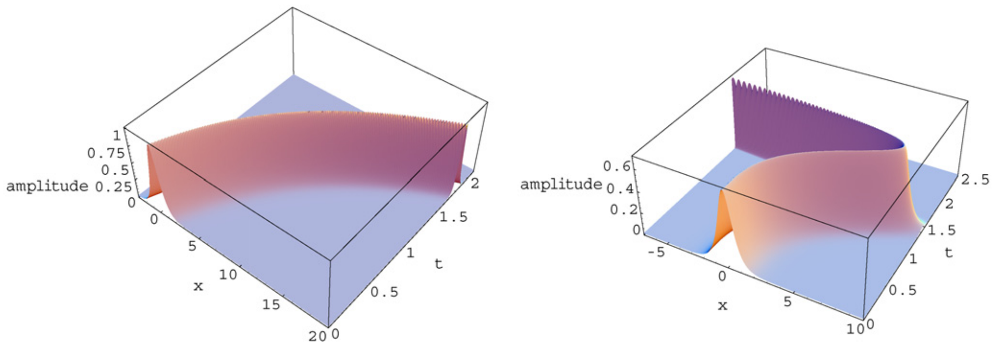

Figure 2. Propagating localized solitary wave solutions for $g = cn(x, t)$, where $\gamma(t) = 3t^2$, $\kappa = -24$, $A_1 = 1$, $P = 1$, $Q = -1$ and $m = 1$; left: $\alpha(t) = 0$ and right: $\alpha(t) = -t^9$.

$\alpha(t) = -3t^3\cos(t^3)$. Figure 2 depicts the same solution when $\gamma(t)$ and $\alpha(t)$ have polynomial time dependence. One clearly sees that the temporal variations of γ and α can effectively modulate and control the propagation of the solitons.

Case II ($g = sn(w, m)$)

We now study the cases where $g = sn(\omega(x, t))$, for which the solution corresponds to the negative MKdV equation:

$$u = A_1 sn\left(\sqrt{\frac{A_1^2\kappa}{6m^2}}\left[x - \int\left[\alpha(t) - \frac{\gamma(t)(m^2+1)A_1^2\kappa}{6m^2}\right]\mathrm{d}t\right]\right). \tag{17}$$

Here, $\gamma(t) > 0$, $Q = m^2 > 0$ and $P = -(m^2 + 1) < 0$.

For $m^2 = 1$, we have the kink-type solitary wave solution

$$u = A_1\tanh\left(\sqrt{\frac{A_1^2\kappa}{6}}\left[x - \int\left[\alpha(t) - \frac{\gamma(t)A_1^2\kappa}{3}\right]\mathrm{d}t\right]\right). \tag{18}$$

Figure 3 depicts the kink solution in the presence of time-dependent dispersion. The soliton motion can be controlled through the external parameters.

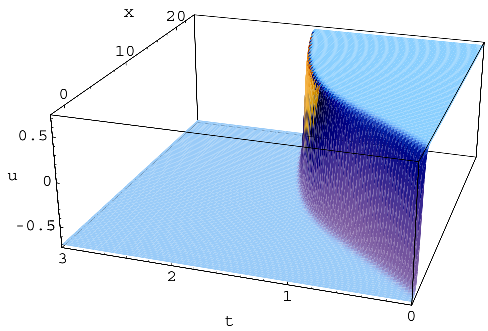

Figure 3. Kink-type solitary wave solution of the MKdV equation, where $\gamma(t) = \cos(t)$, $\alpha(t) = 5t^4$, $\kappa = -48$, $A_1 = 1$, $P = -2$, $Q = 1$ and $m = 1$.

Case III ($g = dn(w, m)$)

We get exact solitary wave solution of the negative MKdV equation, in the case when $g = dn(w(x, t))$:

$$u = A_1 dn\left(\sqrt{\frac{-A_1^2\kappa}{6}}\left[x - \int\left[\alpha(t) - \frac{\gamma(t)(2-m^2)A_1^2\kappa}{6}\right]\mathrm{d}t\right]\right). \tag{19}$$

Here, $\gamma(t) < 0$ and $P = (2 - m^2) > 0$ and $Q = -1$. For $m^2 = 1$, we get bell-shaped solitary wave solution as

$$u = A_1 dn\left(\sqrt{\frac{-A_1^2\kappa}{6}}\left[x - \int\left[\alpha(t) - \frac{\gamma(t)A_1^2\kappa}{6}\right]\mathrm{d}t\right]\right). \tag{20}$$

In conclusion, the MKdV equation with time varying coefficients has solitary waves solutions, provided the temporal variations of the coefficients are of the form given in the text. The temporal variation of these parameters allows effective control of the solitary wave profile. These continuous waves and localized solutions can be made to accelerate. The amplitude and widths are not modulated by the distributed coefficients. The induction of time-dependent u_x term allows us to control the motion of the solitons more efficiently.

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